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Lepton Flavor Violation - Theory

Outline

1. Sources of Lepton Flavor Violation (LFV) and Implications for LFV K Decays:
   - neutrino masses
   - generational symmetries
   - extended technicolor
   - leptoquarks
   - supersymmetry
2. $K^+ \to \pi^+ \mu^\pm e^\mp$ and $K_L^0 \to \pi^0 \mu^\pm e^\mp$
3. $K_L^0 \to \mu^\pm e^\mp$
4. $K^+ \to \mu^+ \nu_h$
5. $|\Delta L| = 2$ Decays: $K^+ \to \pi^- \mu^+ \mu^+$
6. Conclusions

Sources of Lepton Flavor Violation: Neutrino Masses and Mixing

At present, there is strong evidence for neutrino oscillations and hence for neutrino masses and lepton mixing. This mixing violates lepton family (flavor) number and is the first confirmed physics beyond the Standard Model. Recall evidence:

- solar neutrino data from Homestake chlorine exp., Kamiokande, SAGE, GALLEX, SuperKamiokande, and SNO, which demonstrated flavor conversion by measuring both the CC reaction $\nu_e d \to epp$ and the NC reaction $\nu_\ell d \to \nu_\ell pp$;
- atmospheric neutrino data from Kamiokande, IMB, Soudan-2, and especially SuperK, with confirmation from MACRO;
- K2K - terrestrial long-baseline ν_μ disappearance exp.;
- KamLAND reactor $\bar{\nu}_e$ disappearance exp.

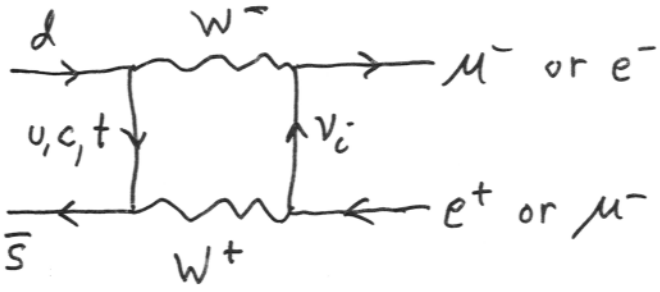
This data is consistent with oscillations among three neutrinos, and yields for the mass and mixing parameters (with $\Delta m^2_{jk} = m(\nu_j)^2 - m(\nu_k)^2$):

- $|\Delta m^2_{32}| \simeq 2 \times 10^{-3}$ eV^2 and $\sin^2 2\theta_{23} \simeq 1$, i.e. $\theta_{23} \simeq 45°$, from fits to atmospheric neutrino oscillations, consistent with K2K
- $\Delta m^2_{21} \simeq 7 \times 10^{-5}$ eV^2 and $\tan^2 \theta_{12} \simeq 0.4$, $\theta_{12} \simeq 33°$ from fits to solar neutrino data and KamLAND
- $\sin^2 2\theta_{13} \lesssim 0.17$ from CHOOZ reactor experiment with above inputs; value of CP-violating phase δ unknown

Most natural theoretical explanations lead to hierarchical neutrino masses. With normal hierarchy, infer $m(\nu_3) \simeq \sqrt{|\Delta m^2_{32}|} \simeq 0.04$ eV, $m(\nu_2) \simeq \sqrt{\Delta m^2_{21}} \simeq 0.008$ eV.

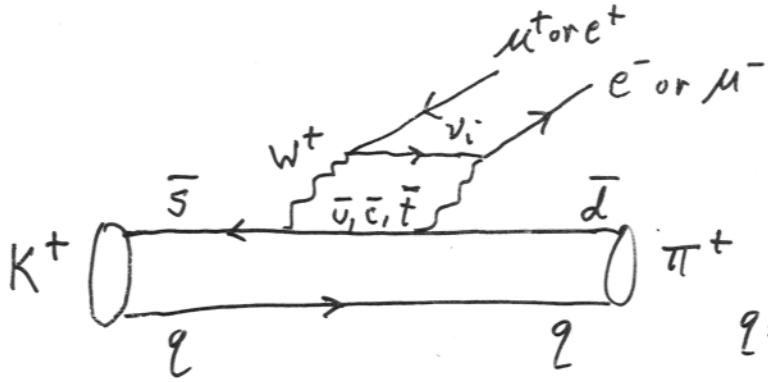
Because of the very small neutrino masses, this violation of lepton family produces effects that are much too small to be seen directly via LFV decays such as $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, $K^+ \to \pi^+\mu^\pm e^\mp$, $K_L \to \mu^\pm e^\mp$, and $\mu \to e$ conversion. This is because of the leptonic GIM mechanism in the SM extended to include neutrino masses, which suppresses the rates for these processes by $(\Delta m^2_\nu/m^2_W)^2 << 1$.

Graphs for LFV K decays in SM extended to include ν masses + lepton mixing
(e.g. B.W. Lee, R.S.)



$$\Rightarrow K_L^0 \to \mu^\pm e^\mp$$

+ $(s \leftrightarrow d)$



$q = u \Rightarrow K^+ \to \pi^+ \mu^\pm e^\mp$

$q = d \Rightarrow K_L^0 \to \pi^0 \mu^\pm e^\mp$

Can also consider models with right-handed charged currents and hence no strict leptonic GIM mechanism, but effects are still much smaller than current levels of sensitivity.

Sources of Lepton Flavor Violating K Decays: Generational Symmetries

The reason for fermion generations is not understood at present. A possibly appealing way to unify them is to hypothesize a generational gauge symmetry such as U(3). Given upper bounds on such processes as $K^+ \to \pi^+\mu^+e^-$, $K_L \to \mu^\pm e^\mp$, etc., it follows that the generational gauge symmetry is broken, and the associated vector bosons are quite massive.

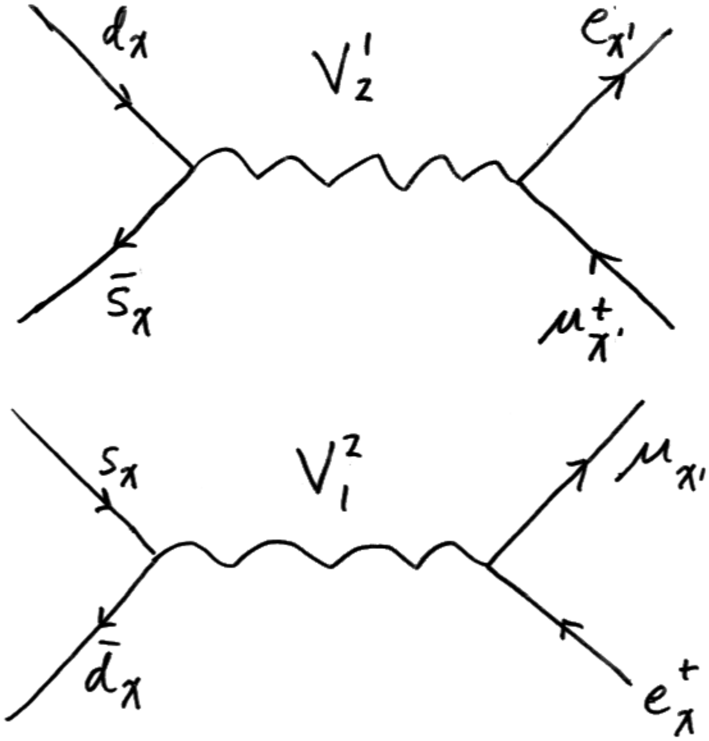
Below, we will show how such a generational gauge symmetry group occurs as part of a fundamental theory (extended technicolor) and is important for explaining fermion masses and mixing, but the possibility of a generational gauge symmetry is more general than ETC.

If we let the left- and right-handed chiral components of quarks and leptons transform according to fundamental representations of this U(3), then some basic transitions would include, e.g.,

$$s_\chi \to d_\chi + V_1^2, \quad \mu_\chi \to e_\chi + V_1^2, \quad \chi = L, R$$

where the gauge boson V_k^j has generational indices j, k. This then leads to the following decays:

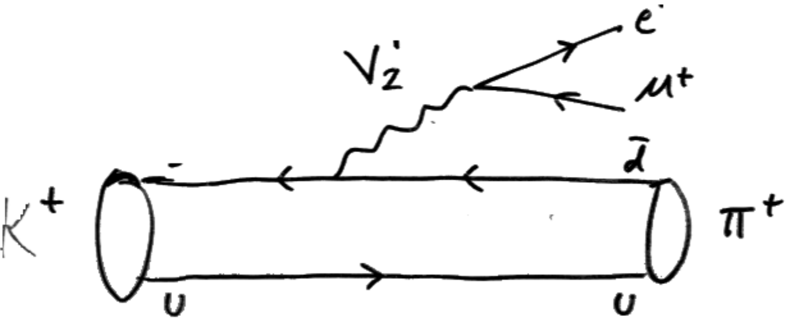
$\chi, \chi' = LL$



$\Rightarrow \; K_L^0 \to \mu^\pm e^\mp$

involves hadronic matrix element of axial-vector cu nt

$\langle 0 | J_{G,A}^\lambda | K_L^0 \rangle$

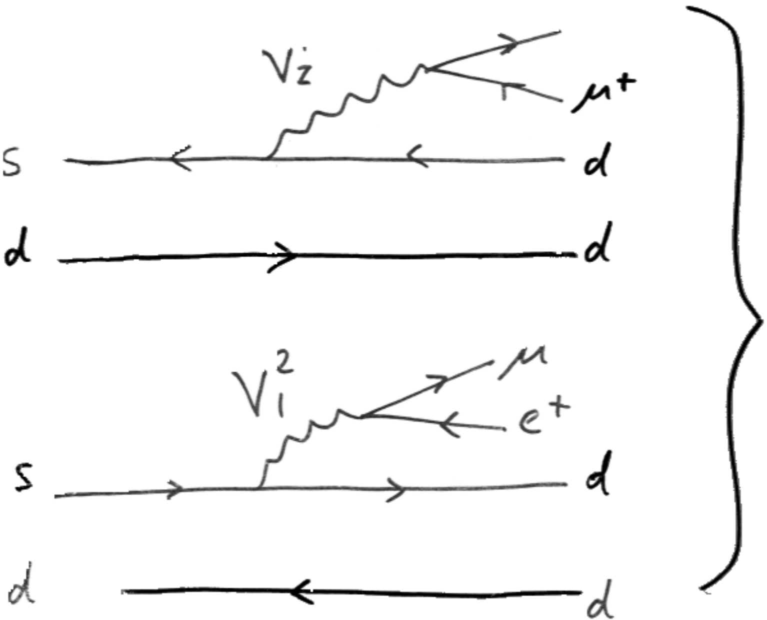


$\Rightarrow \; K^+ \to \pi^+ \mu^+ e$

invol hadr m trix elements of vector cur ent

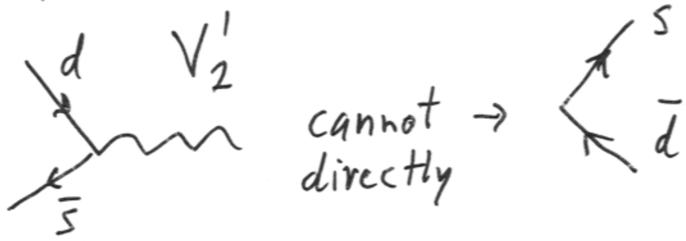
$\langle \pi^+ \; J_G{}^\lambda{}_V \; K^+ \rangle$

$\langle \pi \; J_G \; V \; K_L \rangle$

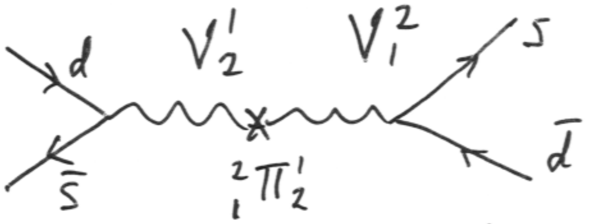


$\Rightarrow \; K_L^0 \to \pi \; \mu^\pm e^\mp$

Constraint from K^0-$\bar{K}^0$ mixing: to leading order the $d\bar{s} \to s\bar{d}$ transition requires $V_2^1 \to V_1^2$ which strongly suppresses the amplitude in several ETC models:

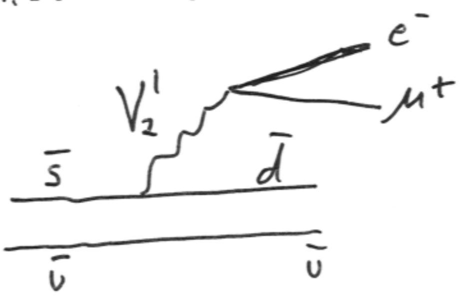


need



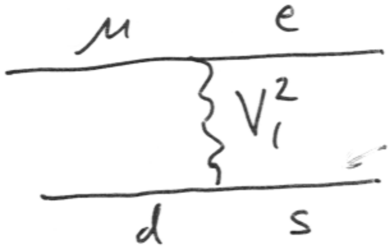
so $$(\Delta m_K)_{ETC} \sim \frac{g_{ETC}^2}{M_1^2}\,{}^2_1\Pi^1_2\,\frac{1}{M_1^2} \ll \frac{g_{ETC}^2}{M_1^2}$$

Relation with LFV in, e.g. μ-e conversion: mechanisms are somewhat different - e.g.



$\Rightarrow \quad K^+ \to \pi^+ \mu^+ e^-$

but this V_2^1 - exchange cannot produce μ-e conv.



← produces second-generation quark

Different fermion mixing angles contribute to these LFV K decays + μ-e conversion

Interaction eigenstates with respect to the U(3) generational gauge symmetry are not, in general, same as mass eigenstates, but to begin, neglect this mixing effect. Then the effective Hamiltonian for $K^+ \to \pi^+\mu^+e^-$ is

$$\mathcal{H}_G = \left(\frac{g_G}{\sqrt{2}}\right)^2 \frac{1}{M_G^2}[\bar{s}\gamma_\lambda(c_{q,V} - c_{q,A}\gamma_5)d][\bar{e}\gamma^\lambda(c_{\ell,V} - c_{\ell,A}\gamma_5)\mu] + h.c.$$

where gauge boson V_2^1 mediating this transition has mass M_G. Recall that for $K_{\ell 3}^+$, usual effective weak Hamiltonian is

$$\mathcal{H}_{wk} = V_{us}^*\left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^2}[\bar{s}_L\gamma_\lambda d_L][\bar{e}_L\gamma^\lambda\mu_L] + h.c.$$

where $f_{L,R} = \frac{(1\mp\gamma_5)}{2}f$ and $g^2/(8m_W^2) = G_F/\sqrt{2}$. Normalizing w.r.t. $K_{\mu 3}$ and using flavor SU(3) relation $\langle\pi^+|U_-|K^+\rangle = \sqrt{2}\langle\pi^0|V_-|K^+\rangle$, we get

$$\frac{BR(K^+ \to \pi^+\mu^+e^-)}{BR(K^+ \to \pi^0\mu^+\nu_\mu)} \quad \frac{16}{|V_{us}|^2}\left(\frac{g_G}{g}\right)^4\left(\frac{m_W}{M_G}\right)^4 |c_{q,V}|^2\left(|c_{\ell,V}|^2 + |c_{\ell,A}|^2\right)$$

Here $\langle\pi^+|J_{G,\lambda}|K^+\rangle$ arises from vector part of the hadronic current $J_{G,\lambda}$. This yields

$$M_G \geq (150\ \text{TeV})\left|\frac{g_G}{g}\right||c_{q,V}|^{1/2}(|c_{\ell,V}|^2+|c_{\ell,A}|^2)^{1/4}\left[\frac{10^{-12}}{B(K^+ \to \pi^+\mu^+e^-)}\right]^{1/4}$$

As an example, consider an ETC theory with vectorial ETC gauge couplings to SM quarks and charged leptons, so $c_{q,V} = c_{\ell,V} = 1$, $c_{q,A} = c_{\ell,A} = 0$, set $M_G = M_1$ and take $\alpha_G = 1/2$, since ETC theory is strongly interacting at the 100 TeV mass scale. Because of this strong coupling, the lowest-order, tree-level contribution to this and other LFV decays is only a rough estimate.

Substituting current limit from BNL E865 (at 90 % CL),

$$BR(K^+ \to \pi^+\mu^+e^-) < 2.8 \times 10^{-11}$$

yields the lower bound

$$M_1 \gtrsim 250 \quad \text{TeV}$$

This illustrates the great power of these limits on LFV K decays; current limits imply sensitivity to new mass scales of 100's of TeV, far above mass scale directly probed by LHC.

E865 has analyzed additional data, which will reduce the limit on $BR(K^+ \to \pi^+\mu^+e^-)$ by about a factor of 2 (M. Zeller, private communication), hence increase M_1 limit by $\sim 2^{1/4}$ to $M_1 \gtrsim 300$ TeV.

Mixing effects (e.g., in ETC theories) can also produce the $|\Delta G| = 2$ decay $K^+ \to \pi^+\mu^-e^+$, so this is also of interest. For $|\Delta G| = 2$ decay, current bound from BNL E865:

$$BR(K^+ \to \pi^+\mu^-e^+) < 5.2 \times 10^{-10}$$

For $K_L^0 \to \pi^0 \mu^\pm e^\mp$,

$$\frac{BR(K_L^0 \to \pi^0 \mu^\pm e^\mp)}{BR(K_L^0 \to \pi^+ \mu^- \bar{\nu}_\mu)} = \frac{4}{V_{us}|^2} \left(\frac{g_G}{g}\right)^4 \left(\frac{m_W}{M_G}\right)^4 |c_{q,V}|^2 \left(|c_{\ell,V}|^2 + |c_{\ell,A}|^2\right)$$

which yields

$$M_G \geq (175\ \text{TeV})\left|\frac{g_G}{g}\right||c_{q,V}|^{1/2}(|c_{\ell,V}|^2+|c_{\ell,A}|^2)^{1/4}\left[\frac{10^{-12}}{B(K_L^0 \to \pi^0 \mu^\pm e^\mp)}\right]^{1/4}$$

With the same illustrative ETC theory as above, using current limit from KTeV (M. Corcoran, hep-ex/0402033)
Bellavance thesis

$$B(K_L^0 \to \pi^0 \mu^\pm e^\mp) \leq 3.3 \times 10^{-10}$$

gives $M_1 \gtrsim 160$ TeV.

Good theoretical motivation for further experiments to reduce upper limits on $BR(K^+ \to \pi^+ \mu^\pm e^\mp)$ and $BR(K_L^0 \to \pi^0 \mu^\pm e^\mp)$ since these would increase lower bound on generation-changing vector boson masses in an interesting range and would further constraint FCNC effects in SUSY (more below). auxiliary part of KOPIO or CKM ? or E949 ?

For $K_L^0 \to \mu^\pm e^\mp$ decay, normalizing to $K^+ \to \mu^+ \nu_\mu$, we have

$$\frac{BR(K_L^0 \to \mu^\pm e^\mp)}{BR(K^+ \to \mu^+ \nu_\mu)} = \frac{4}{V_{us}|^2}\left(\frac{g_G}{g}\right)^4\left(\frac{m_W}{M_G}\right)^4\left(\frac{\tau_{K_L^0}}{\tau_{K^+}}\right)|c_{q,A}|^2(|c_{\ell,V}|^2+|c_{\ell,A}|^2)$$

Here $\langle 0|J_{G,\lambda}|K^0_L\rangle$ arises from axial vector part of hadronic current $J_{G,\lambda}$. This yields

$$\geq (310\,\mathrm{TeV})\left|\frac{g_G}{g}\right||c_{q,A}|^{1/2}(|c_{\ell,V}|^2+|c_{\ell,A}|^2)^{1/4}\left[\frac{10^{-12}}{B(K^0_L \to \mu^\pm e^\mp)}\right]^{1/4}$$

In ETC models with vectorial couplings of ETC gauge bosons to quarks and charged leptons, the amplitude at tree level and including ETC loops yields $c_{q,A} = 0$. Electroweak loop corrections would produce nonzero, but small $c_{q,A}$. However, other models of generational physics could have $c_{q,A} \sim O(1)$, and the upper bound on this decay strongly constrains such models.

Current limit from BNL E871:

$$BR(K^0_L \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}$$

Other LFV K decay modes respecting total lepton no. include, e.g., $K^0_L \to \pi^0\pi^0\mu^\pm e^\mp$, $K^0_L \to \pi^\pm\pi^\mp\mu^\pm e^\mp$, but these involve higher-multiplicity final states and phase space suppression.

In K decays, can also search for the LFV decay $\pi^0 \to \mu^\pm e^\mp$, but for a given source of LFV, expect $BR(\pi^0 \to \mu^\pm e^\mp) << BR(K^0_L \to \mu^\pm e^\mp)$ since $\Gamma_{\pi^0}/\Gamma_{K^0_L} = 6 \times 10^8$.
Current limit from KTeV: $BR(\pi^0 \to \mu^\pm e^\mp) < 7.85 \times 10^{-10}$.

Lepton Flavor Violation in Theories with Dynamical Electroweak Symmetry Breaking

First, recall some motivations for these theories.

- The Standard Model (SM) does not explain electroweak symmetry breaking (EWSB); it simply puts it in by hand, by making $\mu^2 < 0$ for the coefficient $\mu^2\phi^\dagger\phi$ in the Higgs potential.
- Neither the SM nor grand unified theories explain the number of generations, which are put in via replication of the fermion representations of the gauge group.
- Fermion masses in the SM break the electroweak symmetry. The SM accomodates, but does not explain, these masses and the associated mixing, via Yukawa couplings to the Higgs. The associated Yukawa couplings for e, u, d are of order $10^{-5} - 10^{-6}$ without any underlying reason.
- The Higgs sector of the SM has a hierarchy problem, quadratic instability to large loop corrections, sensitivity to high-scale physics.

Dynamical EWSB theories have the potential to provide

- an explanation of EWSB, via the formation of a bilinear condensate of technifermions $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_R \rangle$. This condensate is formed because the technifermions are subject to a new exact strong (confining) interaction, technicolor (TC) (Weinberg, Susskind). Analogy with formation of $\langle \bar{q}q \rangle$

condensate in QCD, which also breaks EW symmetry, since in both cases the left-handed and right-handed fermions transform differently under the EW gauge group.

- an explanation for fermion generations and the generational hiearchy, since these generations are gauged and the gauge symmetries are dynamically broken
- an explanation for fermion masses and mixing, which are dynamically generated
- a solution to the gauge hierarchy problem, since there is no Higgs

One can recall that in both of two major previous cases of symmetry breaking in which one used a scalar field as part of a phenomenological approach, the microscopic physics actually involved bilinear fermion condensates:

- Ginzburg-Landau approach to superconductivity using complex scalar field; microscopic BCS theory involved dynamical formation of Cooper pair condensate
- Gell-Mann Levy σ model for spontaneous chiral symmetry breaking in hadronic physics, in which the SχSB was manifested by $\langle\sigma\rangle \neq 0$; in the actual underlying QCD theory, the SχSB is due to the dynamical formation of a $\langle\bar{q}q\rangle$ condensate

Could these previous examples be a hint as to the underlying physics beyond the SM?   Perhaps.

A common version of TC has one SM family of technifermions, U_χ, D_χ, N_χ, and E_χ, transforming as the fundamental representation of the TC gauge group and having usual SM quantum numbers for color and weak isospin and hypercharge.

The TC theory is asymptotically free, so that as the energy decreases, its gauge coupling increases, and, at the scale Λ_{TC} this coupling is strong enough for the formation of a nonzero technicondensate $\langle \bar{F}F \rangle$. Since the technifermions are massless in the underlying theory, this condensate spontaneously breaks the associated global TC chiral symmetries and leads to Nambu-Goldstone bosons which are absorbed by the W and Z, giving them masses $m_W = m_Z \cos\theta_W \simeq g\Lambda_{TC}/2$. Hence, $\Lambda_{TC} \simeq 300$ GeV.

The $m_W = m_Z \cos\theta_W$ mass relation in TC follows because $\langle \bar{F}F \rangle$ transforms as $I = 1/2$, $|Y| = 1$ under weak isospin and hypercharge, just as the SM Higgs does.

To give masses to SM fermions, the technicolor theory is embedded in an extended technicolor (ETC) theory, which communicates the EWSB to these fermions (Eichten, Lane, Dimopoulos, Susskind).

In order to give sufficiently large masses to fermions while suppressing flavor-changing neutral currents (FCNC) enough, modern (E)TC theories are designed to have a slowly running ("walking") gauge coupling over a range of energies (Holdom, Appelquist, Yamawaki..).

To satisfy constraints on FCNC processes, ETC vector bosons must have large masses, especially for those that couple to the first two generations. These masses can arise from self-breaking of the ETC symmetry, which requires that ETC be a strongly coupled, chiral gauge theory.

Consider models based on an ETC gauge group SU(N_{ETC}). Since this group contains both generations and technicolor,

$$N_{ETC} = N_{gen.} + N_{TC}$$

The choice $N_{TC} = 2$ is motivated for several reasons; it

- minimizes the TC contributions to the S parameter measuring heavy fermion loop contributions to the Z propagator
- can yield an approximate infrared fixed point and associated walking behavior
- makes possible a mechanism for explaining light neutrinos in an ETC framework (Appelquist, Shrock)

Combining $N_{TC} = 2$ with the number of SM generations, $N_{gen} = 3$, one has $N_{ETC} = 5$ in this class of models, i.e., $G_{ETC} = \mathrm{SU}(5)_{ETC}$

Consider a class of ETC theories such that $[G_{ETC}, G_{SM}] = 0$.

Since the ETC theory is a chiral gauge theory, there are initially no fermion masses; hence all fermion masses are generated dynamically.

This theory is constructed to be asymptotically free, so that as the energy decreases, the ETC gauge coupling increases. The ETC self-breaking occurs in stages, e.g., $\Lambda_1 \simeq 10^3$ TeV, $\Lambda_2 \simeq 10^2$ TeV, and $\Lambda_3 \simeq 4$ TeV. These are determined by the strong dynamics and are required to produce the correct magnitude of fermion masses.

Dynamical EWSB theories are subject to a number of tight constraints from precision electroweak measurements and bounds on FCNC processes. The latter are of particular interest here.

Constraints from precision electroweak data

$$S \simeq \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

Naive perturbative evaluation: each extra EW doublet gives contribution

$$\Delta S \simeq \frac{N_{TC}}{6\pi}\left[1 - Y \ln\left(\frac{m_{1/2}^2}{m_{-1/2}^2}\right)\right]$$

where subscript is weak T_3. In one-family TC, have

$$\begin{pmatrix} U^{ia} \\ D^{ia} \end{pmatrix} \qquad \begin{pmatrix} N^i \\ E^i \end{pmatrix}$$

where $\{i\}$ TC indices, $a =$ color indices

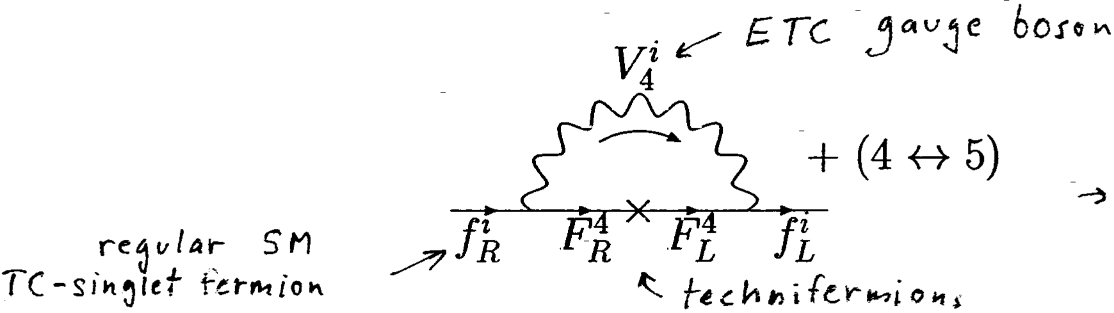
Nondegeneracy of U, D and of N, E masses can reduce TC contribution to S, as constrained by T parameter ($\rho = \alpha T$).

Since technifermions are strongly interacting at scale $E \sim m_Z$, one cannot use perturbative estimates, and one does not have reliable methods to calculate TC contributions to S and T. Non-QCD-like behavior (walking) means that one cannot reliably try to scale up from QCD.

N.B.: Efforts to perform precision electroweak fits with oblique corrections have been complicated considerably by the NuTeV anomaly

Mass Generation for Quarks and Charged Leptons

Recall dynamical ETC mass generation mechanism for quarks and charged leptons. For rough estimate, consider one-loop diagram shown.



$+ (4 \leftrightarrow 5)$ $\rightarrow \bar{f}_{Li} f_R^i$

One-loop graphs contributing to the mass term $\bar{f}_{i,L} f_R^i$ where $1 \le i \le 3$.

This yields $m_{fi} \propto g_{ETC}^2 \eta_i \Lambda_{TC}^3 / M_i^2$ where $M_i \sim (g_{ETC}/4)\Lambda_i$ is the mass of the ETC gauge bosons that gain mass at scale Λ_i and g_{ETC} is the running ETC gauge coupling at this scale. The factor η_i is a enhancement factor for a theory with walking:

$$\eta_i = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha(\mu))\right]$$

If TC has walking up to a scale Λ_w and over this interval the anomalous dimension $\gamma \simeq 1$, then $\eta_i \simeq \Lambda_i/\Lambda_{TC}$ for $\Lambda_i \le \Lambda_w$, and Λ_w/Λ_{TC} for $\Lambda_i \ge \Lambda_w$. Hence, for a theory with walking up to Λ_1,

$$m_{fi} \propto \frac{\Lambda_{TC}^2}{\cdot}$$

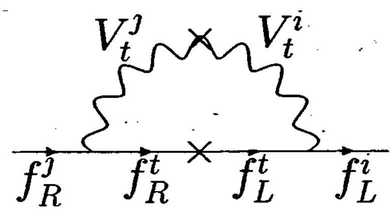
and, e.g., for walking up to Λ_w,

$$m_{fi} \propto \frac{\eta_i \Lambda_{TC}^3}{\Lambda_i^2}$$

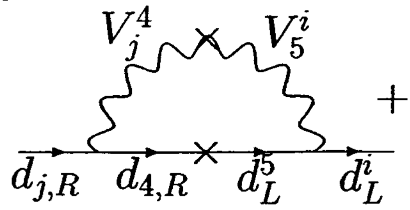
Other variants use dif. repr. of ETC

Here the left- and right-handed chiral components of the SM fermions and the corresponding technifermions are assigned to the same (fundamental) representation of the ETC group.

There are mixings among the interaction eigenstates of the ETC gauge bosons to form vector boson mass eigenstates. These involve mass mixings of the form $V_t^j \to V_t^i$, where $i, j \in \{1, 2, 3\}$ and $t \in \{4, 5\}$. Insertions of these on ETC gauge boson lines lead to off-diagonal elements of the fermion mass matrices via diagrams like the one shown below:



Another class of ETC models assigns left- and right-handed components of down-type (techni)quarks to relatively conjugate representations of the ETC group, and similarly for the charged leptons, in order to provide for intra-generational mass splittings $m_t >> m_b, m_\tau, m_c >> m_s, m_\mu$ (Appelquist, Piai, Shrock). For this class of models, the fermion mass matrices arise from the diagram below, and others involving more ETC gauge boson lines (recall that the ETC coupling is strong)



$+ (4 \leftrightarrow 5)$

In both of these classes of models the ETC interactions dynamically determine the elements of the fermion mass matrices, and hence CKM mixing. Denote SM fermion mass terms as

$$\mathcal{L}_m = -\sum_f \sum_{j,k=1}^{3} \bar{f}_{j,L} M_{jk}^{(f)} f_{k,R} + h.c.$$

where $f \quad u, d, e$ for up-, down-type quarks and charged leptons. $M^{(f)}$ can be diagonalized by

$$U_L^{(f)} M^{(f)} U_R^{(f)\ -1} \quad M_{diag.}^{(f)}$$

Hence, the ETC interaction eigenstates f are mapped to mass eigenstates f_m via

$$f_\chi = U_\chi^{(f)\ -1} f_{m,\chi}$$

for $\chi = L, R$; e.g., $(u^1, u^2, u^3)_L$ is mapped via $U_L^{(u)}$ to $(u_m^1, u_m^2, u_m^3)_L \equiv (u, c, t)_L$, etc. The CKM quark mixing matrix that enters in the charged weak current $J_\lambda = \bar{u}_{j,m,L} \gamma_\lambda V_{jk} d_{k,m,L}$ is $V = U_L^{(u)} U_L^{(d)\ \dagger}$.

Parametrize the transformation matrices $U_\chi^{(f)}$, $\chi = L, R$, as

$$U_\chi^{(f)} = P_\alpha^{(f)\chi} O^{(f)} P_\beta^{(f)\chi}$$

where

$$P_\alpha^{(f)\chi} \quad \mathrm{diag}(e^{i\alpha_1^{(f)\chi}}, e^{i\alpha_2^{(f)\chi}}, e^{i\alpha_3^{(f)\chi}}), \quad P_\beta^{(f)\chi} \quad \mathrm{diag}(e^{i\beta_1^{(f)\chi}}, e^{i\beta_2^{(f)\chi}}, e^{i\alpha_3^{(f)\chi}})$$

$$O^{(f)} = R_{23}(\theta_{23}^{(f)\chi}) R_{13}(\theta_{13}^{(f)\chi}) R_{12}(\theta_{12}^{(f)\chi})$$

where R_{jk} is the rotation in the jk subsector.

For fermions f_χ, $\chi = L, R$, transforming as 5's of $\mathrm{SU}(5)_{ETC}$, the basic coupling to the ETC gauge bosons is

$$\mathcal{L} = g_{ETC} \bar{f}_{j,\chi} (T_a)^j_k (V_a)^\lambda \gamma_\lambda f^k_\chi$$

where the T_a, $a = 1, ..24$ are the generators of $\mathrm{SU}(5)_{ETC}$. Rewrite this in terms of ETC shift operators and corresponding gauge bosons for $j \neq k$, and diagonal generators for $j = k$,

$$\begin{aligned}
T_{24} \equiv T_{d1} &\quad (2\sqrt{10})^{-1}\mathrm{diag}(\quad 4, 1, 1, 1, 1) \\
T_{15} \equiv T_{d2} &= (2\sqrt{6})^{-1}\mathrm{diag}(0, -3, 1, 1, 1) \\
T_8 \equiv T_{d3} &= (2\sqrt{3})^{-1}\mathrm{diag}(0, 0, -2, 1, 1) \\
& \quad (1/2)\mathrm{diag}(0, 0, 0, -1, 1)
\end{aligned}$$

When $\mathrm{SU}(5)_{ETC}$ breaks to $\mathrm{SU}(4)_{ETC}$ at the highest scale Λ_1, the first generation of fermions splits off and the ETC gauge bosons in the coset $\mathrm{SU}(5)_{ETC}/\mathrm{SU}(4)_{ETC}$, i.e., V^1_j and $(V^1_j)^\dagger = V^j_1$ with $2 \leq j \leq 5$, and V_{d1}, gain mass $M_1 \sim \Lambda_1$.

Similarly, when $\mathrm{SU}(4)_{ETC}$ breaks to $\mathrm{SU}(3)_{ETC}$ at scale Λ_2, the second generation of fermions splits off and V_j^2 and V_2^j with $3 \leq j \leq 5$, and V_{d2}, gain mass $M_2 \sim \Lambda_2$

Final breaking $\mathrm{SU}(3)_{ETC} \rightarrow \mathrm{SU}(2)_{TC}$ at Λ_3; V_j^3 and V_3^j with $4 \leq j \leq 5$, and V_{d3}, gain mass $M_3 \sim \Lambda_3$.

Define

$$\mathcal{V} = \begin{pmatrix} -\frac{2V_{d1}}{\sqrt{10}} & \frac{V_2^1}{\sqrt{2}} & & \frac{V_3^1}{\sqrt{2}} \\ \frac{V_1^2}{\sqrt{2}} & \frac{V_{d1}}{2\sqrt{10}} & \frac{3V_{d2}}{2\sqrt{6}} & \frac{V_3^2}{\sqrt{2}} \\ \frac{V_1^3}{\sqrt{2}} & \frac{V_2^3}{\sqrt{2}} & & \frac{V_{d1}}{2\sqrt{10}} + \frac{V_{d2}}{2\sqrt{6}} - \frac{V_{d3}}{\sqrt{3}} \end{pmatrix}$$

The coupling of the ETC gauge bosons to the mass eigenstates of fermions is then

$$\mathcal{L}_{int} = g_{ETC} \sum_{f,j,k,\chi} \bar{f}_{j,\chi} \gamma_\lambda (\mathcal{V}^\lambda)^j_k f^k_\chi \equiv g_{ETC} \sum_{f,j,k,\chi} \bar{f}_{m,j,\chi} \gamma_\lambda (A^\lambda)^j_k f^k_{m,\chi}$$

where

$$A^\lambda \equiv U_\chi^{(f)} \mathcal{V}^\lambda U_\chi^{(f)\ -1}$$

The angles in $U_\chi^{(f)}$ are constrained by measurements of regular muon decay, $K^0 - \bar{K}^0$ and $B_d^0 - \bar{B}_d$ mixing, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, together with limits on $B_s^0 - \bar{B}_s$ and $D^0 - \bar{D}^0$ mixing and various LFV processes such as $K^+ \rightarrow \pi^+ \mu^\pm e^\mp$, $K_L \rightarrow \mu^\pm e^\mp$, $B_d \rightarrow \mu^\pm e^\mp$, $\mu^+ \rightarrow e^+ e^+ e^-$, and $\mu \rightarrow e$ conversion. The analysis of the ETC contributions to these processes focuses on the induced local dimension-6 four-fermion operators.

ETC gauge boson exchange diagrams and fermion mixing also contribute to (diagonal and transition) dimension-5 magnetic and electric dipole moments for leptons and quarks, and, for quarks, also chromo-magnetic and chromo-electric dipole operators.

The lepton dipole moments contribute to processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$, and μ and e electric dipole moments. The quark dipole operators contribute to processes such as $b \to s\gamma$, and neutron and atomic EDM's. Study of constraints, taking into account this mixing: Appelquist, Piai, Shrock, hep-ph/0401114 (PLB), to appear, and with Christensen. For example, constraints on ETC contributions to $\mu \to e\gamma$ yield bounds

$$|\theta_{13}^{(e)L}\theta_{23}^{(e)R}|, \ |\theta_{13}^{(e)R}\theta_{23}^{(e)L}| \lesssim 10^{-5}$$

so the observed large lepton mixing arises from the neutrino sector in these models.

Because of the fermion mixing, a process such as $K^+ \to \pi^+\mu^+e^-$ that only involves fermions of generations $j = 1, 2$ can couple to V_{d3} with a mass M_3 much smaller than the mass scale M_1 of V_2^1; the diagrams thus involve much less propagator suppression but are suppressed by small fermionic mixing angles, and the analyses of this and related processes set~~s~~ bounds on these mixing angles. Taking $\Lambda_3 \simeq 4$ TeV as above and performing a small-angle expansion, we get

$$|\theta_{13}^{(d)\chi}\theta_{23}^{(d)\chi}\theta_{13}^{(e)\chi'}\theta_{23}^{(e)\chi'} \lesssim 10^{-4}$$

for χ, χ' $\quad LL, LR, RL, RR$.

As illustration, consider $\theta^{(d)}_{jk} \simeq \theta_{jk,CKM}$ so that $\theta^{(d)}_{13} \simeq 0.004$ and $\theta^{(d)}_{23} \simeq 0.04$; then the quark mixing angles give a factor $|\theta^{(d)}_{13}\theta^{(d)}_{23}| \simeq 1.6 \times 10^{-4}$ and the small leptonic mixing angles suppress this further, satisfying the bound.

Also because of fermion mixing and the fact that flavor-changing processes can proceed via exchange of V_{d3}, one gets $K^+ \to \pi^+\mu^-e^+$ as well as $K^+ \to \pi^+\mu^+e^-$ decays.

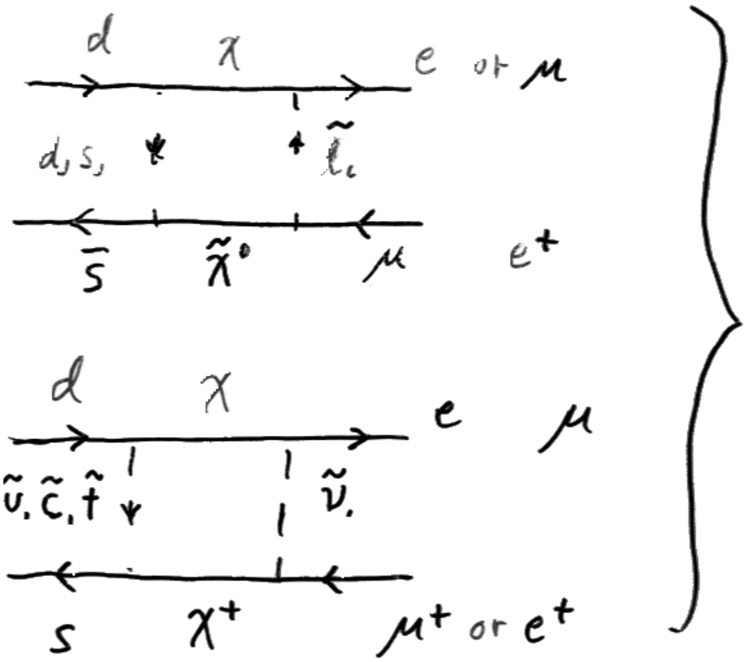
LFV K Decays in Supersymmetric Extensions of SM

Since the transformations that diagonalize the squark and slepton mass squared matrices are not, in general, the same as the transformations that diagonalize the quark and lepton matrices, one can get FCNC effects in supersymmetric extensions of the SM.

Many studies on this. One can consider both R-parity conserving and R-parity violating scenarios. SUSY contributions to LFV K decays are constrained by limits on $K^0 - \bar{K}^0$ mixing, $\mu \to e\gamma$, $\mu - e$ conversion, etc. At BNL, with its planned sensitivity down to $R_{\mu e} \sim 10^{-16}$ or below, the MECO experiment should probe a significant range of SUSY parameter space; similarly with the PRISM exp. The future $\mu \to e\gamma$ experiment MEG at PSI will also be useful.

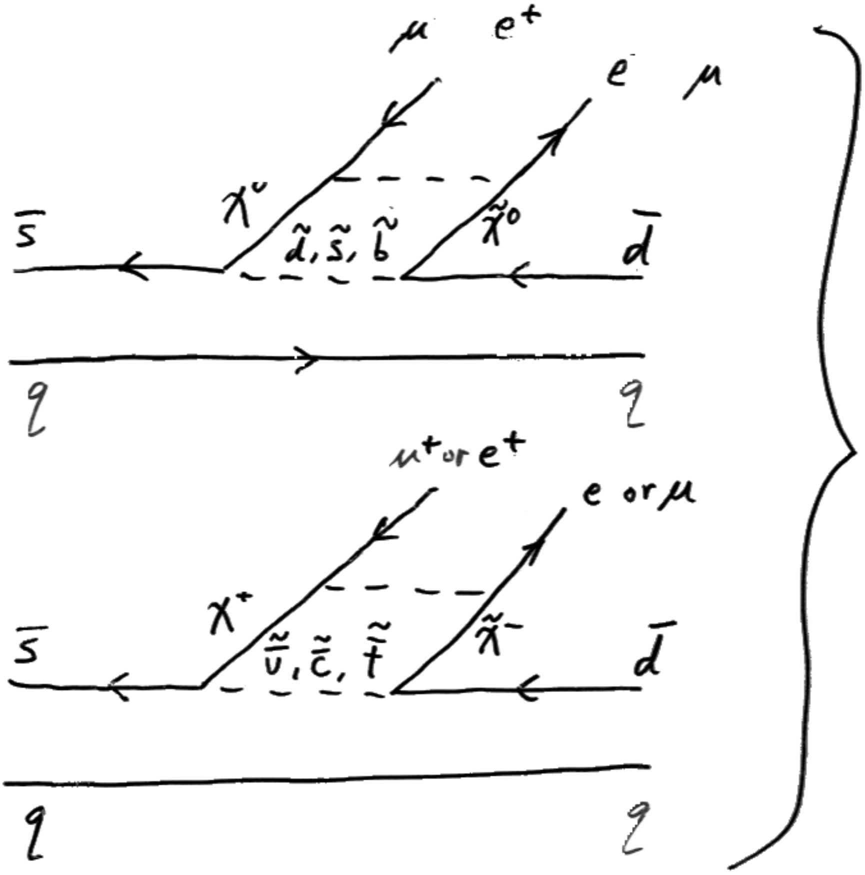
Since LFV K decay experiments involve both leptonic and quark couplings to SUSY particles, they can be complementary to searches for $\mu \to e\gamma$, $\mu - e$ conversion.

Examples of graphs that give LFV K decays:



$\Rightarrow \quad K_L \to \mu^\pm e^\mp$

(d ↔ s



for q u $\Rightarrow$

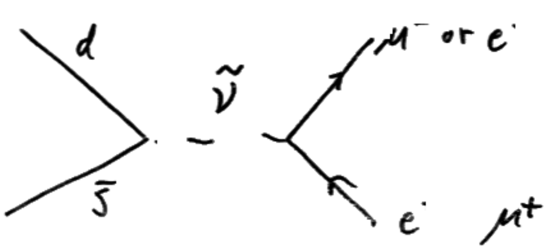
$K^+ \to \pi^+ \mu^\pm e^\mp$

for q d $\Rightarrow$

$K_L^0 \to \pi\, \mu^\pm e^\mp$

n R pari y v ating SUSY can get other graphs

e.g



(from CERN Kaon Physics Working Gp. Rept. hep-ph/0107046)

$tg\beta = 10$ $tg\beta = 20$

$K_L^0 \to \mu^\pm e^\mp$

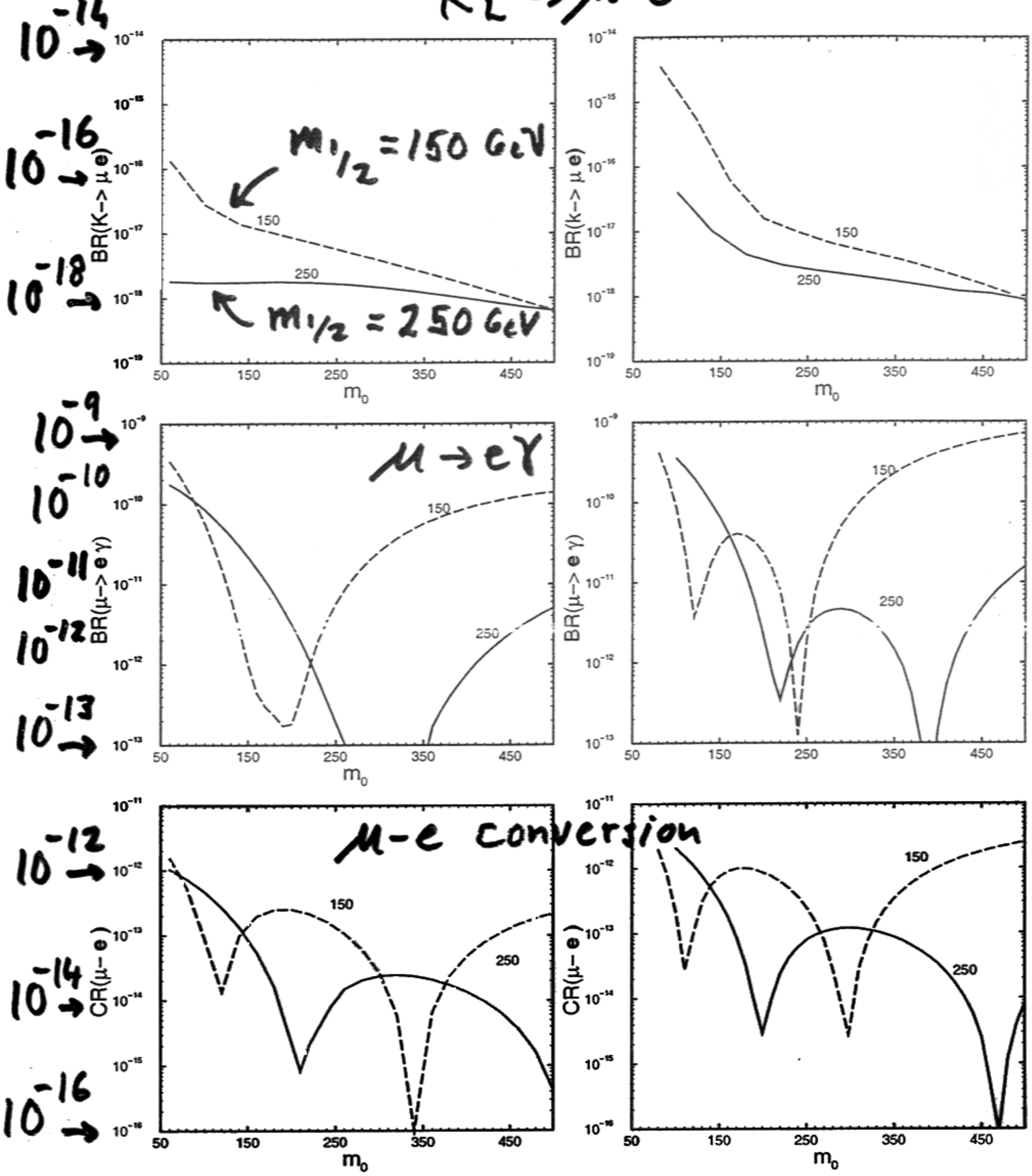


Figure 2: *Illustrative predictions for $BR(K \to \mu e)$, $BR(\mu \to e\gamma)$ and $BR(\mu - e)$ for different values of $\tan\beta = 10$ (left column), 20 (right column) and $m_{1/2} = 150$ (dashed lines), 250 GeV (solid lines), as functions of m_0 (in GeV).*

Neutrino Masses in Models with Extended Gauge Symmetries

(Appelquist + Shrock, PRL 90, 201801 (2003))

We have succeeded in constructing similar models explaining light neutrino masses in theories with dynamical symmetry breaking of extended strong-electroweak gauge groups that have appealing features going beyond those of the SM. The first such group is

$$G_{LR} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$$

in which the usual fermions of each generation transform as

$$(3,2,1)_{1/3,L}\ , \quad (3,1,2)_{1/3,R}$$
$$(1,2,1)_{-1,L}\ , \quad (1,1,2)_{-1,R}$$

The gauge couplings are defined via the covariant derivative

$$D_\mu = \partial_\mu - ig_3 \mathbf{T}_c \cdot \mathbf{A}_{c,\mu} - ig_{2L} \mathbf{T}_L \cdot \mathbf{A}_{L,\mu} - ig_{2R} \mathbf{T}_R \cdot \mathbf{A}_{R,\mu} - i(g_U/2)(B - L)\mathbf{U}_\mu$$

Here the electric charge is given by the elegant relation

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

where $B =$ baryon no., $L =$ lepton number. Given experimental limits on right-handed charged currents and an associated W_R, and on extra Z''s, $\mathrm{SU}(2)_R$ must be broken at a scale Λ_{LR} well above the electroweak scale. Similarly for $U(1)_{B-L}$;

$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ at Λ_{LR}

The second extended gauge group is

$$G_{422} = \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

(Pati, Salam, Mohapatra, Senjanovic..)

with the usual fermions transforming as

$$(4, 2, 1)_L \ , \quad (4, 1, 2)_R$$

provides a higher degree of unification since

It unifies quarks and leptons in the $(4, 2, 1)_L$ and $(4, 1, 2)_R$ representations for each generation; e.g., for the first-generation, these are

$$\begin{pmatrix} u^a & \nu_e \\ d^a & e \end{pmatrix}_{L,R}$$

- It combines $\mathrm{U}(1)_{B-L}$ and $\mathrm{SU}(3)_c$ (in a maximal subgroup) in the Pati-Salam group $\mathrm{SU}(4)_{PS}$ and hence relates g_U and g_3 . Denoting the generators of $\mathrm{SU}(4)_{PS}$ as $T_{PS,i}$, $1 \leq i \leq 15$, with

$$T_{PS,15} \quad \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

and setting $U_\mu = A_{PS,15,\mu}$, one has $(B - L)/2 \quad \sqrt{2/3} T_{PS,15}$ and hence

$$\frac{g_U^2}{g_{PS}^2} = \frac{3}{2}$$

- It quantizes electric charge:

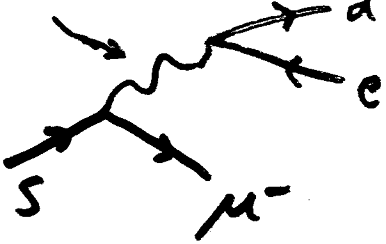
$$Q = T_{3L} + T_{3R} + \sqrt{2/3}\,T_{PS,15} = T_{3L} + T_{3R} + (1/6)diag(1,1,1,-3)$$

In these theories with Pati-Salam unification, one can get LFV K decays, and the upper limits on the BR's for these decays imply that the scale Λ_{PS} at which SU(4)$_{PS}\times$ SU(2)$_R$ breaks to SU(3)$_c\times$ U(1)$_Y$ is $\gtrsim O(100)$ TeV.

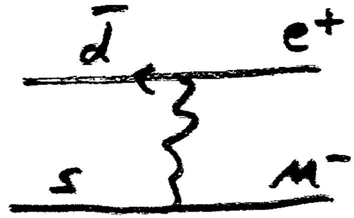
Examples of graphs that give LFV K decays:

reps. $\begin{pmatrix} u & \nu_e \\ d & e \end{pmatrix}$, $\begin{pmatrix} c & \nu_\mu \\ s & \mu \end{pmatrix}$

PS gauge boson — color triplet, charge 2/3 ("leptoquark")



or



$$\Rightarrow \quad K_L^0 \to \mu^\pm e^\mp$$

$$K^+ \to \pi^+ \mu^+ e^-$$

$$K_L^0 \to \pi^0 \mu^\pm e^\mp$$

$K^+ \to \ell^+ \nu_h$

As an auxiliary analysis of data from high-statistics K^+ decay experiments, one could set improved bounds on the possibility of the emission of heavy neutrinos, $K^+ \to \ell^+ \nu_h$, where $\ell = \mu, e$. Search for anomalously low ℓ^+ momentum, which tags the mass of $m(\nu_h)$; set upper limit on the coupling $|U_{\ell,h}|$. Old searches at KEK set upper limits on $|U_{\mu,h}|^2$ down to $\sim 10^{-6}$ for $m(\nu_h)$ in range of 100-300 MeV.

Origin: in addition to the usual three electroweak-doublet neutrinos, many theories include some number k of electroweak-singlet neutrinos. In a GUT-type seesaw, the latter have masses $\sim M_{GUT}$.

However, in theories with dynamical EWSB, one gets light neutrinos by a different kind of seesaw that does not involve any GUT-scale masses (Appelquist and Shrock, PLB 548, 204 (2002); PRL 90, 201801 (2003); Appelquist, Piai, Shrock, PRD D69, 015002 (2004)). Here, $m_\nu \sim m_D^2/m_R$, where m_D is a strongly suppressed Dirac neutrino mass generated at loop level, of order 1-10 KeV, and m_R is Majorana neutrino mass also generated at loop level, of order $m(\nu_h) \sim 0.1 - 10$ GeV. Given their decay rates, these intermediate-mass neutrinos can be consistent with bounds from astrophysical and cosmological constraints.

(In these ETC theories, there are also much heavier Majorana neutrinos, with masses up to 10^3 TeV, which are not relevant here.)

The full neutrino mixing matrix is thus $(3+k) \times (3+k)$, although the usual 3×3 lepton mixing matrix mapping the mass eigenstates ν_j, $j = 1, 2, 3$ to the the EW-doublet ν_ℓ, $\ell = e, \mu, \tau$, is very close to being unitary. There is nonzero, but very small, mixing of the mass eigenstate ν_h in ν_ℓ, $\ell = e, \mu, \tau$. The mixing is roughly

$$\theta \sim \frac{m_D}{m_R} \sim \frac{(1 - 10 \text{ KeV})}{(\sim 1 \text{ GeV})} \sim 10^{-5} - 10^{-6}$$

so $|U_{\mu,h}|^2 \sim 10^{-11}$, very small. Also, $m(\nu_h)$ could be too large to occur in K decay.

But from a general phenomenological viewpoint, and as shown by the ETC model, such decays might occur and might be observable, motivating an analysis of current and future data.

$|\Delta L| = 2$ K DECAYS

The existing upper limits on $0\nu 2\beta$ decay are the best on $|\Delta L| = 2$ decays involving ee. Using the upper bound on $\langle m_\nu \rangle$, one can obtain an upper limit on processes such as $K^+ \to \pi^- e^+ e^+$ much less than currently directly measured BR. limit.

Consider also upper limits on some meson and baryon decays involving μ's that violate total lepton number (by $|\Delta L| = 2$): $K^+ \to \pi^- \mu^+ \mu^+$, $K^+ \to \pi^- \mu^+ e^+$, and $\Xi^- \to p \mu^- \mu^-$.

L. Littenberg and R. S., Phys. Rev. Lett. 68, 443 (1992), Phys. Rev. D46, R892 (1992): from retroactive data analysis, set first upper limits (90 % CL):

$$BR(K^+ \to \pi^- \mu^+ \mu^+) < 1.5 \times 10^{-4}$$

$$BR(\Xi^- \to p \mu^- \mu^-) < 3.7 \times 10^{-4}$$

(these had not been searched for previously) Also, in the PRL, set an indirect upper limit on $BR(K^+ \to \pi^- \mu^+ e^+)$ using the fact that the leptonic part of the amplitude for this decay is related by crossing to that for the process $\mu^- + Ti \to e^+ + Ca$, and the existing upper limit on
$\sigma(\mu^- + Ti \to e^+ + Ca)/\sigma(\mu^- + Ti \to capture)$.

- proposed dedicated search for the K^+ modes in the BNL K decay program. This has now been carried out as a byproduct of the E865 experiment searching for $K^+ \to \pi^+ \mu^+ e^-$, yielding the new limits (hep-ex/0006003) → PRL 85, 2877 (2000)

$$BR(K^+ \to \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}$$

$$BR(K^+ \to \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}$$

Also $BR(K^+ \to \pi^- e^+ e^+) < 6.4 \times 10^{-10}$.

One possible source is intermediate-mass neutrinos, but as illustrated by the ETC model, the relevant mixing angles are probably too small to observe these decays. Another possible source is R-parity violating SUSY, but again, expect BR well below the 10^{-10} level (Littenberg and RS, 2000)

But, as with the $K^+ \to \ell^+ \nu_h$ search, if one has the opportunity, it is worthwhile to improve existing search limits.

Conclusions

Very good upper limits have been set on lepton-flavor violating K decays by BNL E865 and E871, and by KTeV. These illustrate well the power of searches for rare/forbidden processes in constraining physics beyond the SM; for example, they limit masses of generation-changing gauge boson masses in ETC theories to be in the 100's of TeV region, far beyond the scale of new physics that will be directly studied at LHC.

Although lepton family no. violation has already been established by evidence for neutrino oscillations, LFV searches in K decays are sensitive to different aspects of this phenomenon, as is illustrated by the examples of generation-changing gauge interactions, ETC, leptoquarks, and SUSY.

There is thus motivation for improving these limits on LFV K decays. This is admittedly challenging, given the excellent bounds already obtained.

LFV K decays are complementary to $\mu \to e\gamma$ and $\mu - e$ conversion since they involve couplings to quarks. Specific scenarios for beyond-SM physics can be further constrained by (i) future LFV K decay searches, (ii) ongoing and future precision studies of the B and K systems (in the latter case, $K^+ \to \pi^+ \nu\bar{\nu}$ and KOPIO), (iii) MEG, MECO, and PRISM, (iv) neutrino oscillation studies and deep underground experiments; and (v) high-energy colliders.